Intuitive Control of Dynamic Simulation Using Improved Implicit Constraint Enforcement

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Abstract. Geometric constraints are imperative components of many dynamic simulation systems to effectively control the behavior of simulated objects. In this paper we present an improved first-order implicit constraint enforcement scheme to achieve improved accuracy without significant computational burden. Our improved implicit constraint enforcement technique is seamlessly integrated into a dynamic simulation system to achieve desirable motions during the simulation using constraint forces and doesn't require parameter tweaking for numerical stabilization. Our experimental results show improved accuracy in maintaining constraints in comparison with our previous first-order implicit constraint method and the notable explicit Baumgarte method. The improved accuracy in constraint enforcement contributes to the effective and intuitive motion control in dynamic simulations. To demonstrate the wide applicability, the proposed constraint scheme is successfully applied to the prevention of excessive elongation of cloth springs, the realistic motion of cloth under arduous collision conditions, and the modeling of a joint for a rigid body robot arm.

1 Introduction

One of the primary goals of a dynamic simulation is to create physically realistic motions and the intuitive control of a dynamic simulation plays a key role in accomplishing the intended realism. Maintaining geometric constraints with utmost accuracy can be an effective tool to control the behavior of simulated objects in many applications including physically based simulation, character animation, and robotics. Recently we have established the first-order implicit constraint enforcement to effectively enforce geometric constraints [2]. This paper presents the improved first-order implicit constraint enforcement that extends our previous work to efficiently and robustly administrate the motion of objects. The enhanced accuracy in constraint enforcement enables us to improve the motion control in many applications.

Hard springs and penalty forces [13, 14] are often used as an alternative to geometric constraints but they may create a stiff numerical system and consequently the simulator may have to use a small integration time step to avoid numerical instability. Furthermore, the accuracy of the intended behavior control is not warranted since the

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choice of coefficients can greatly alter the associated object behavior. Constrained dynamics using Lagrange Multipliers are well studied and widely used to enforce geometric constraints. Baumgarte constraint stabilization technique has been successfully used for many applications [3, 4, 7, 8, 9, 15] to reduce the constraint errors and stabilize the constrained dynamic system. Barzel and Barr [8] introduced dynamic constraints to control behaviors of rigid bodies using geometric constraints. Platt and Barr [9] presented reaction constraints and augmented Lagrangian constraints to flexible models. Witkin and Welch [7] applied the explicit Baumgarte method into non-rigid objects to connect them and control the trajectory of motion. Baraff [4] used linear time Lagrange multiplier method to solve constraint system. Ascher and Chin [11] explicated the inherent drawbacks of Baumgarte stabilization method and introduced constraint stabilization techniques. Cline and Pai [6] introduced post-stabilization approach for rigid body simulation which uses stabilization step to compensate the error of integration for each time step. However, it requires additional computational load to reinstate the accuracy and it may cause the loss of natural physical motions of object under complicated situations since the constraint drift reduction is performed independently from the conforming dynamic motions.

The rest of this paper is organized as follows: the brief overview of constraint enforcement is elaborated in section 2. Section 3 elucidates a description of improved first-order implicit constraint enforcement scheme. In section 4 we provide the explanation for our simulated examples and experimental results of our method.

2 Constraint Enforcement

Two main concerns of constraint enforcement to create tractable and plausible motions are physical realism and the controllability of simulation. Our primary contribution of this paper is the enhanced accuracy in constraint enforcement using the improved first-order implicit constraint management scheme which can be applied to any dynamic simulations – character animation, cloth simulation, rigid body simulation, and so on. Without proper control terms or constraints, the dynamic simulations may generate abnormal or undesirable motions. Under this circumstance, it is considered that an accurate and robust constraint management plays an important role in controlling the motion of dynamic simulations. This paper describes a method to achieve exact desired behaviors without any discordant motions of object by employing accurate and effective geometric constraints.

To achieve certain behaviors in a physically-based simulation, the desirable geometric restrictions should be intuitively reflected into the dynamic system. Most constraint-based dynamic systems [1, 4, 6, 7, 8] convert geometric restrictions into constraint forces to maintain the restrictions and they are blended with other external forces. Any relations or restrictions which can be represented as algebraic equations can be applied as constraints in the dynamic simulation system. For instance, animators can enforce various types of constraints to intuitively control desirable initial environments or modify the motions of objects during simulation: distance constraints (node-node or node-arbitrary point), angle constraints, parallel constraints, and inequality constraints [1].

The explicit Baumgarte method [3, 4, 7, 8] uses a classic second order system, similar to a spring and damper, to enforce and stabilize the constraints. This method helps in enforcing constraints within the framework of Lagrange multipliers method, but it requires ad-hoc constant values to effectively stabilize the system. Finding proper problem specific coefficients is not always easy. In order to overcome this problem, we proposed the implicit constraint enforcement approach [2]. It provides stable and effective constraint management with same asymptotic computational cost of explicit Baumgarte method. Moreover, the explicit Baumgarte method is conditionally stable while the implicit method is always stable independent from the step size. This paper is extension of our previous work to improve the accuracy of constraint enforcement. The computational solution is detailed in the implicit constraint using improved first-order scheme section.

3 Implicit Constraint Using Improved First-Order Scheme

One of the most popular methods for integrating a geometric constraint into a dynamic system is to use Lagrange multipliers and constraint forces. The constraint-based formulation using Lagrange multipliers results in a mixed system of ordinary differential equations and algebraic expressions. We write this system of equations using 3n generalized coordinates, q, where n is the number of discrete masses, and the generalized coordinates are simply the Cartesian coordinates of the discrete masses.

$$q = \left[x_1 y_1 z_1 x_2 y_2 z_2 \cdots x_n y_n z_n\right]^T$$

Let $\Phi(q(t),t)$ be the constraint vector made up of m components each representing an algebraic constraint. The constraint vector is represented mathematically as

$$\Phi\left(q(t),t\right) = \left[\Phi^1\left(q(t),t\right)\Phi^2\left(q(t),t\right)\cdots\Phi^m\left(q(t),t\right)\right]$$

where the Φ^i are the individual scalar algebraic constraint equations. We write the system of equations

$$\begin{aligned} M\ddot{q} + \Phi_q^T \lambda &= F^A \\ \Phi\left(q(t),t\right) &= 0 \end{aligned} \tag{1}$$

where F^A are applied, gravitational and spring forces acting on the discrete masses, M is a 3nx3n diagonal matrix containing discrete nodal masses, λ is a mx1 vector containing the Lagrange multipliers and Φ_q is the mx3n Jacobian matrix, and subscript q indicates partial differentiation with respect to q.

We previously proposed an implicit first-order constraint enforcement scheme to effectively control cloth behavior [2]. That implicit method was first order and was implemented the following way. The equation of motion (equation (1)) along with the kinematics relationship between q and $\dot{q}$ are discretized as

$$\dot{q}(t+\Delta t) = \dot{q}(t) - \Delta t M^{-1}\Phi_q^T\left(q(t),t\right)\lambda + \Delta t M^{-1}F^A\left(q(t),t\right) \tag{2}$$

$$q(t+\Delta t) = q(t) + \Delta t\dot{q}(t+\Delta t) \tag{3}$$

The constraint equations written at new time thus they are treated implicitly

$$\Phi\left(q(t+\Delta t), t+\Delta t\right) = 0 \tag{4}$$

Equation (4) is now approximated using a truncated, first-order Taylor series

$$\Phi(q(t),t)+\Phi_q(q(t),t)(q(t+\Delta t)-q(t))+\Phi_t(q(t),t)\Delta t=0$$

Note that the subscripts q and t indicate partial differentiation with respect to q and t, respectively. Eliminating $q(t+\Delta t)$ results in the following linear system with λ the remaining unknown.

$$\Phi_q(q(t),t)M^{-1}\Phi_q^T(q(t),t)\lambda=\frac{1}{\Delta t^2}\Phi(q(t),t)+\frac{1}{\Delta t}\Phi_t(q(t),t)$$
$$+\Phi_q(q(t),t)\left(\frac{1}{\Delta t}\dot{q}(t)+M^{-1}F^A(q(t),t)\right)$$

Note that the coefficient matrix for this implicit method is the same as the coefficient matrix for the Baumgarte method. This system is solved for the Lagrange multipliers then equations (2) and (3) are used to update the generalized coordinates and velocities.

In what follows we describe a scheme that while formally first-order, contains a correction that improves the accuracy of the scheme without significant computational cost. This scheme may be described as using a second order mid-point rule for the momentum equation while solving the constraint equations implicitly using a first order linearization. The first step calculates all variables including the Lagrange multiplier at the half-time step.

$$\dot{q}(t+\Delta t/2)=\dot{q}(t)-\frac{\Delta t}{2}\left(M^{-1}\Phi_q^T(q(t),t)\lambda-M^{-1}F^A(q(t),t)\right) \tag{5}$$

$$q(t+\Delta t/2)=q(t)+\frac{\Delta t}{2}\dot{q}(t+\Delta t/2) \tag{6}$$

$$\Phi(q(t),t)+\Phi_q(q(t),t)(q(t+\Delta t/2)-q(t))+\Phi_t(q(t),t)\frac{\Delta t}{2}=0 \tag{7}$$

Again note that the subscript t indicates partial differentiation with respect to t. $q(t+\Delta t/2)$ and $\dot{q}(t+\Delta t/2)$ are eliminated from equations (5) through (7) resulting in the linear system that must be solved to obtain the half-step Lagrange multipliers:

$$\Phi_q(q(t),t)M^{-1}\Phi_q^T(q(t),t)\lambda=\frac{4}{\Delta t^2}\Phi(q(t),t)$$
$$+\Phi_q(q(t),t)\left(\frac{2}{\Delta t}\dot{q}(t)+M^{-1}F^A(q(t),t)\right)$$

Equations (5) and (6) are then used to obtain $q(t+\Delta t/2)$ and $\dot{q}(t+\Delta t/2)$. The second step uses the half-step Lagrange multiplier and evaluates the Jacobian using $q(t+\Delta t/2)$ to form the mid-point step:

$$\dot{q}(t+\Delta t)=\dot{q}(t)-\Delta t M^{-1}\Phi_q^T(q(t+\Delta t/2),t+\Delta t/2)\lambda+\Delta t M^{-1}F^A(q(t),t)$$
$$q(t+\Delta t)=q(t)+\Delta t\dot{q}(t+\Delta t/2)$$

This scheme is formally first order as the linearized constraint equation (equation (7)) is solved only to first order. Despite this theoretical first order accuracy the scheme exhibits improved accuracy while requiring solution of the same linear system as the first order scheme described above.

To integrate non-constrained portion of system, the second-order explicit Adams-Bashforth method [10] is used to predict the next time status of the system. The explicit Adams-Bashforth method for solving differential equations is second order and provides better stability than first-order explicit Euler method. The second-order Adams-Bashforth method has the general form:

$$\eta_{i+1} = \eta_i + \frac{1}{2}h\left(3f(x_i,\eta_i) - f(x_{i-1},\eta_{i-1})\right)$$

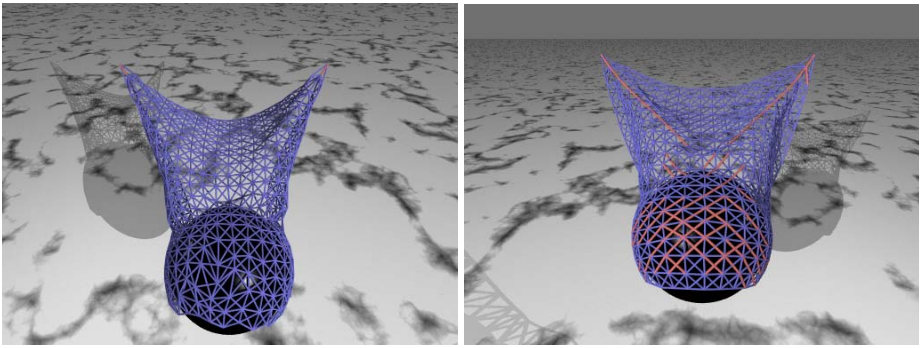
This equation is integrated into ODE (Ordinary Differential Equation) system as a force computation to calculate next status of velocity and then we can estimate the next status of position.

$$\dot{q}(t+\Delta t) = \dot{q}(t) + \Delta t M^{-1}\left(3F^A(q(t),t) - F^A\left(q(t-\Delta t),t\right)\right)$$
$$q(t+\Delta t) = q(t) + \Delta t\dot{q}(t+\Delta t)$$

4 Applications and Experimental Results

One good example of the constraint enforcement for controlling motion is "super-elastic" effect [5] in cloth simulation. Most clothes have their own properties for elongation. Stiff springs can be a candidate solution for this problem, but there still exists numerical instability for linear or non-linear springs when we apply a large integration time step. Provot [5] applied the distance reduction using dynamic inverse constraints for over-elongated springs. However, this approach ignores physical consequences of moving nodes instantly and it may cause loss of energy or original dynamic behavior of motions. In addition, the ordering of node displacement is critical when the internal meshing structures are complex and collision is involved. To prevent excessive elongation of cloth, our approach replaces the springs with implicit constraints when the elongations of springs are over a certain threshold. In our method, adaptive constraint activation and deactivation techniques are applied to reduce computational burden of solving a larger linear system. For instance, only when springs are over elongated, these springs are replaced with implicit constraints. When the constraint force is working to resist the shrinkage, our system deactivates these constraints into springs.

We applied our new technique to a cloth simulation to prevent over-elongation of cloth springs. A piece of cloth is initially in a horizontal position and falling down due to gravity. A relatively heavy ball is moving toward the middle of the falling cloth. Figure 1 illustrates the different motion of cloth without (a) and with (b) constraint enforcement. In figure 1 (a), some springs are excessively stretched and they cause unstable motion of cloth simulation. However, figure 1 (b) shows plausible and stable motion of cloth using implicit constraint enforcement. Although a moving ball hits the middle portion of falling cloth which abruptly creates severe collision forces to specific parts of cloth, an animation [12] using implicit constraint enforcement



(a) Without constraint enforcement (b) With constraint enforcement

Fig. 1. Mesh view of simulated motion of a cloth patch on the collision between a moving ball and the freely falling cloth under gravity. The cloth is hung by two fixed point-node constraints and the implicit constraint enforcement is used to prevent the excessive elongation of cloth springs.

shows realistic behavior of the cloth and an animation in figure 1 (a) using only springs includes unstable and odd motions of cloth.

To test the accuracy of the constraint enforcement and measure the total amount of constraint drift during simulation, we have tested our improved first-order implicit constraint enforcement system with a robotic link connected with revolute joints. Figure 2 shows 5-link robot arm falling down under gravity. Initially the robot links are in horizontal position and one end of it is fixed. We recorded the constraint error over each time step which is obtained by accumulated absolute value of difference between each original link distance and current link distance.

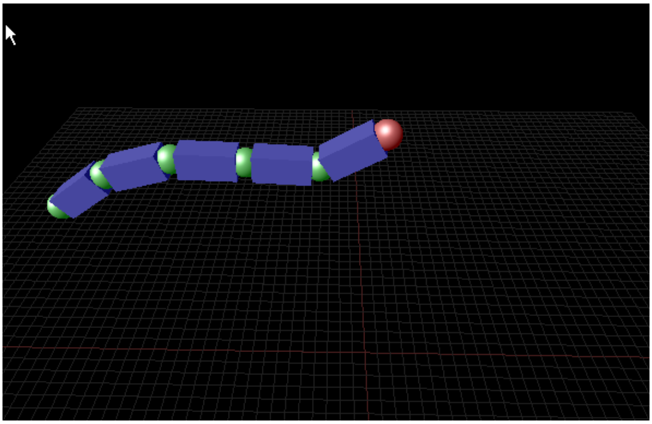


Fig. 2. A simulation of a freely falling 5-link robot arm under gravity. Each Link is constrained using improved implicit constraint enforcement and a red sphere illustrates fixed point-node constraint to hang the robot links.

Figure 3 shows an accumulated experimental constraint error data from the simulation of a falling 5-link robot arm shown in figure 2. We compared implicit constraint enforcement using first-order scheme and the improved first-order scheme and tested these systems using two different categories: integration time step size (0.01 and 0.1 seconds) and mass (1 and 50 unit). Although the accumulated constraint error grows for bigger masses and bigger integration time step sizes, the implicit constraint enforcement using the improved first-order scheme bestows approximately two times higher accuracy. In figure 4, we measure the accumulated constraint error under same condition of figure 2 using explicit Baumgarte method. This graph shows that the implicit constraint enforcement reduces constraint errors effecting comparison with the overall errors of Baumgarte stabilization. The choice of alpha and beta feedback terms has a dominant effect on how well this Baumgarte constraint enforcement performs.

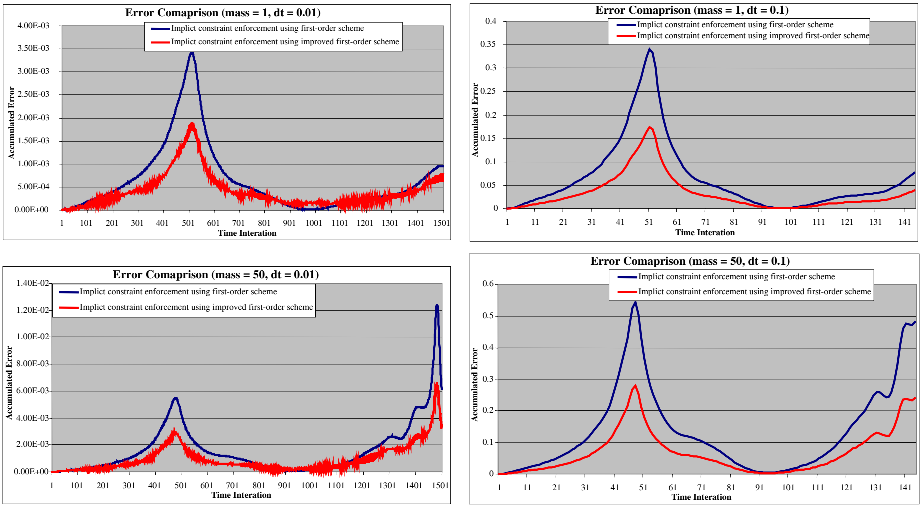


Fig. 3. Accumulated error of implicit constraint enforcement using improved first-order scheme and first-order scheme. Different masses and integration time steps are applied to these systems for comparison.

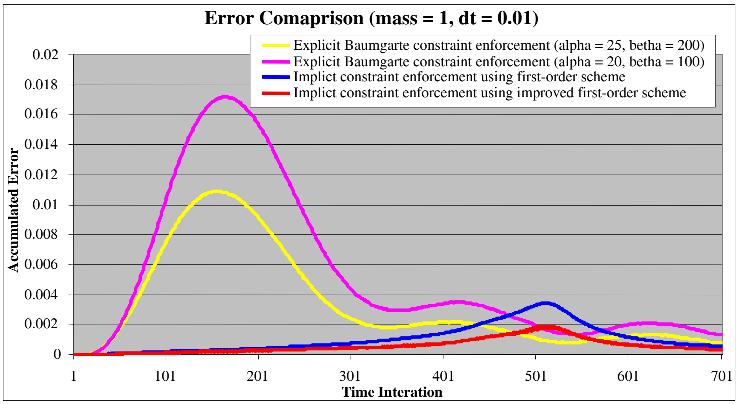


Fig. 4. Accumulated constraint error comparison for explicit and implicit constraint enforcement.

5 Conclusion

This paper describes a new improved first-order implicit constraint enforcement scheme that enhances the accuracy in constraint enforcement. It doesn't require any ad-hoc constants to stabilize the system. While the additional computation cost is relatively minimal, the accuracy of the constraint enforcement is significantly enhanced. This technique can be used to control any dynamic simulation environment using various types of geometric constraints such as angle or distance restrictions. This paper shows how well the control of the over elongation problem of cloth is handled using the implicit constraint enforcement. Adaptive constraint activation and deactivation approach also reduces the computational expense of the numerical system. This paper focuses on the constraint enforcement and minimizing constraint drift with an assumption that a desirable behavior can be represented in geometric constraints. In the future, more studies in formulating a set of proper constraints from intended behaviors of objects and minimizing the total number of active constraints are needed.

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References

1. M. Hong, M. Choi and R. Yelluripati. Intuitive Control of Deformable Object Simulation using Geometric Constraints, Proc. The 2003 International Conference on Imaging Science, Systems, and Technology (CISST' 03), 2003.
2. M. Choi, M. Hong and W. Samuel. Modeling and Simulation of Sharp Creases. Proceedings of the SIGGRAPH 2004 Sketches, 2004.
3. J. Baumgart. Stabilization of Constraints and Integrals of Motion in Dynamical Systems. Computer Methods in Applied Mechanics. 1:1 36. 1972.
4. D. Baraff . Linear-Time Dynamics using Lagrange Multipliers, Computer Graphics Proceedings. Annual Conference Series. ACM Press, 137-146, 1996.
5. X. Provot. Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior. In Graphics Interface. 147-154. 1995.
6. M. B. Cline and D. K. Pai. Post-Stabilization for Rigid Body Simulation with Contact and Constraints. In Proceedings of the IEEE International Conference on Robotics and Automation. 2003.
7. A. Witkin and W. Welch. Fast Animation and Control of Nonrigid Structures. ACM SIGGRAPH 1990 Conference Proceedings. 24. 4. 243-252. 1990.
8. R. Barzel and A. H. Barr. A Modeling System Based on Dynamic Constraints. Computer Graphics Proceedings. Annual Conference Series. ACM Press. 22. 179-188. 1988.
9. J. C. Platt and A. H. Barr. Constraint Methods for Flexible Models. Computer Graphics Proceedings. Annual Conference Series. ACM Press. 22. 4. 279-288. 1988.
10. T. J. Akai. Applied Numerical Methods for Engineers. John Wiley & Sons Inc. 1993.
11. U. R. Ascher, H. Chin and S. Reich. Stabilzation of DAEs and invariant manifolds. Numerische Methematik, 67(2), 131-149, 1994.

12. Computer Graphics Lab. Department of Computer Science and Engineering. University of Colorado at Denver. http://graphics.cudenver.edu
13. A. Witkin, K. Fleischer and A. Barr. Energy Constraints on Parameterized Models. ACM SIGGRAPH 1987 Conference Proceedings. 225-232. 1987.
14. D. Terzopoulos, J. Platt, A. Barr and K. Fleischer. Elastically Deformable Models. ACM SIGGRAPH 1987 Conference Proceedings. 205-214. 1987.
15. D. Metaxas and D. Terzopoulos. Dynamic Deformation of Solid Primitives with Constraints. Computer Graphics. 309-312. 1992.